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Article

The Multi-Period Surgical Scheduling with Capacity Constraint: A Mathematical Modelling Approach

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Abstract: This research proposes a mathematical model for multi-period surgical scheduling problem with capacity constraint over a particular time horizon. The goal is to schedule a list of patients who must undergo various kinds of operations by different eligible hospitals. In particular, each operation must be performed in a particular time period and different operations of one patient can be performed by different eligible hospitals. In addition, each hospital has limited surgery capacity for each time period. The problem is formulated with a multi-objective model using the weighted sum approach of two objectives: minimization of makespan and minimization of total least preference assignment score. The experiment is executed using the simulated data according to the real treatments of cleft lip and palate patients. The results show that the model yield the correct assignment and operation sequence respected to all constraints. Thus, this proposed mathematical model can be further used as smart decision tool in surgical scheduling in hospital network.

Keywords: mathematical model; surgical scheduling; multi-period; capacity constraint; minimization

1. Introduction

One of the most important roles on managerial aspect on health care service is providing prompt health services to patients (Cardoen et al., 2010). Surgical scheduling problem is a complex problem for many operating room administrators. In general, this problem consists of selecting surgeries assigned to operating rooms or hospitals, specify surgical period and the required resources (Silva & de Souza, 2020). Therefore, surgical scheduling is a solution to support decision maker when hospital's management has certain objectives for improving their current systems; for example, improving access, enhancing quality and reducing the cost of health care system or maximize the level of patient satisfaction. However, tradeoff between each objective could be occurred. Increase patient flow and reduce lead time would be benefit for patients but these improvements can lead to an increase in costs (Drupsteen et al., 2013). Recently, Hamid et al. (2019) proposed a multi-objective mathematical model for scheduling of inpatient surgeries includes three objective functions as minimizing total costs related to surgeries, maximizing the level of patient satisfaction according to their priorities and maximizing the compatibility among the surgical team members.

Although hospital administrators can schedule a surgical based on the availability of the operating surgeon and operating room (OR), and suitable timing of the patient (Langer, 1977). Designing surgical scheduling system is important task to enhance healthcare service and ensure an optimal utilization of the costly medical resources, while assuring patient satisfaction. Therefore, recently, a number of researches focusing on patient scheduling has been increased in the aim of maximize patient satisfaction. However, many aspects of the patient experience can affect patient satisfaction with the care received (Langer, 1977).

The surgical scheduling problem can be classified based on planning horizon. Long term planning aims to create and/or upgrade facilities, medium term planning tries to allocate surgical time periods to surgeons, and short-term planning aims to allocate patients to days and times within time periods. In the last minute, some adjustments can be made at short term planning before the schedule is executed. Capacity planning problem is a part of surgical scheduling problem which composes of three components. The first component concerns physical aspect such as the number of rooms and equipment. The second component involves human resource; for examples, the number and type of surgical practices, anesthesiologists, and other OR staff available. The third component is resource availability which includes the number of hours that ORs open and how those hours are parceled out (Donahue et al., 2017). There is a numerous literature on operational capacity planning in hospitals. Obviously, one unit that is of particular interest is the operating room (OR) (May et al., 2011). In most cases, a mixed integer programming (MIP) was used to schedule elective patients for each day of the week from different categories to be admitted into the hospital subject to scarce resources such as beds and operating rooms (Gartner & Kolisch, 2014). Apart from capacity planning mentioned previously, Vissers et al. (2005) solved surgery scheduling problems under stochastic environment based on uncertainty parameters; for example, surgery durations, the arrival of emergency surgeries, and capacity of the surgical intensive care unit.

In this research, surgical scheduling problem for an operation planning level is addressed so that operations of patients can be planned in advance. A novel mathematical model for multi-period surgical scheduling problem with capacity constraint over a time horizon is proposed. The problem is formulated with a multi-objective model using the weighted sum approach of two objectives: minimization of makespan and minimization of total least preference assignment score. Next the computational experiments are executed using LINGO optimization solver for model analysis. Finally, the discussion and conclusion are provided.

2. Materials and Methods

As mentioned earlier, the surgical scheduling problem in this study aims to schedule a list of patients who are supposed to undergo different kinds of surgeries in a particular time horizon. Different operations of each patient can be served at any hospital selected from a set of eligible hospitals. In addition, for each period, different hospitals have different limited capacity to perform a surgery. Therefore, the surgical schedule is planned for each hospital on multiple periods over a particular time horizon. In this section, a mixed-integer programming (MIP) model is proposed to represent the surgical scheduling problem. The decisions in this problem include 1) identifying a list of selected surgeries, 2) assigning each operation of patients to the selected hospital, and 3) identifying the period of the selected surgeries. The model considers two objectives which are minimization of makespan and minimization of total least preference assignment score based on patient location. This model uses the weighted sum approach to combine two objectives into a single objective. The main assumptions of the model are summarized as follows:

- Each patient has different symptoms. Therefore, each patient requires different kinds of operation requirements.
- Some operations can be performed at specific hospitals only.
- Time for operation treatment is a deterministic variable.
- Time for transferring patient between hospitals is not considered.
- There are no preemptions in scheduling.
- All patients have equal priority.

In order to formulate a mathematical model for multi-objective surgical scheduling problem, the notations for indices, parameters, and decision variables are defined as follows.

Indices

i is patient (*i* = 1,…,*n*)
j is treatment or Operation (*j* = 1,…,*q*)
k is hospital (*k* = 1,…,*m*)
t is time period (*t* = 1,…,*u*)

Decision variable

X_{ijkt} = $\begin{cases} 1 \text{ if patient } i \text{ is assigned to hospital } k \text{ for operation } j \text{ in period } t \\ 0 \text{ otherwise} \end{cases}$
Ck_{ij} is treatment completion time of patient *i* operation *j*
Cx_{ijk} is treatment completion time of patient *i* operation *j* at hospital *k*
C_{max} is maximum completion time of all patients
sc_{ijkt} is assignment score of patient *i* operation *j* to hospital *k* in period *t*
Tsc is total score from assignment

Parameters

p_{ijkt} is processing time for treatment patient *i* operation *j* in hospital *k* in period *t*
r_{ij} is ready time for starting operation *j* of each patient *i*
d_{ij} is due date of completing operation *j* of each patient *i*
$capa_{jkt}$ is maximum capacity of hospital *k* that can treat operation *j* in period *t*
w_{ik} is least preference score of patient *i* being treated at hospital *k*
tp_t is time period *t*
e_{ijkt} is hospital eligibility restrictions
$= \begin{cases} 1 & \text{if hospital } k \text{ can treat patient } i \text{ operation } j \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$
a_{ij} is treatment requirement $= \begin{cases} 1 & \text{if patient } i \text{ need to treat an operation } j \\ 0 & \text{otherwise} \end{cases}$

Objective function

$$\text{Minimize } (0.5 * Cmax) + (0.5 * Tsc) \tag{1}$$

Equation (1) shows an objective function of the model. Two objectives which are normalized makespan and normalized least preference score are given equal weight and combined into a single objective.

Constraints

$$\sum_{t=1}^{u} \sum_{k=1}^{m} X_{ijkt} = a_{ij} \;\; ; \;\; \forall i,j \tag{2}$$

Equation (2) ensures that a patient must be treated for each operation by only one hospital for one period (a_{ij} =1) and there is no need to assign patients to other hospitals in any time period if patient i are not required to treat an operation j (a_{ij} =0)

$$X_{ijkt} \leq e_{ijkt} \;\; ; \;\; \forall i,j,k,t \tag{3}$$

Equation (3) specifies that each patient can be assigned to any eligible hospital to treat each of operations.

$$C_{ijk} - C_{ij-1k} \geq \sum_{k=1}^{m} (p_{ijkt} * X_{ijkt}) \;\; ; \forall i,j,k,t \quad ; j \neq 1 \tag{4}$$

Equation (4) is a precedence constraint to ensure that completion time of any operation must be greater than or equal to its ready time plus its processing time.

$$C_{ijk} \geq r_{ij} + P_{ijkt} \;\; ; \forall i,j,k,t \tag{5}$$

$$C_{ijk} \leq d_{ij} \quad ; \forall i,j,k,t \tag{6}$$

Equation (5) and (6) states that the completion time of patient i operation j at hospital k must be between patient's ready time plus processing time of that operation and maximum specified age of a patient to perform that operation.

$$Max\left(Cx_{ijk}\right) = Ck_{ij} \quad ; \ \forall i,j,k \tag{7}$$

$$Ck_{ij} * \ a_{ij} = \sum_{t=1}^{u}\sum_{k=1}^{m} X_{ijkt} * tp_t \quad ; \ \forall i,j \tag{8}$$

Equation (7) and (8) explain the completion time of patient i operation j at hospital k.

$$\sum_{i=1}^{n}\sum_{j=1}^{l} X_{ijkt} \leq capa_{jkt} \ \ ; \ \forall j,k,t \tag{9}$$

Equation (9) is capacity constraint of hospital k that can treat operation j in period t.

$$sc_{ijkt} = X_{ijkt} * w_{ik} \quad ; \forall i,j,k,t \tag{10}$$

$$Tsc = \sum_{i=1}^{n}\sum_{j=1}^{l}\sum_{k=1}^{m}\sum_{t=1}^{u} sc_{ijkt} \tag{11}$$

Equation (10) calculate the assignment score for patient i that being treated operation j in hospital k, and Equation (11) calculate total assignment score for all patients.

$$C_{max} = \ Max\left\{C_{ijk}\right\} \ ; \ \forall i,j,k,j \ ; \ a_{ij} = 1 \tag{12}$$

Equation (12) specify the makespan, Cmax (maximum completion time of all patients).

$$X_{ijkt} \in \{0,1\} \tag{13}$$

Equation (13) specifies that decision variables are binary.

3. Results and Discussions

In this study, numerical example is given to illustrate the solution methods of the proposed surgical scheduling model (Min & Yih, 2010). To make the problem practical, the computational experiments are executed using the simulated data according to the real treatments of cleft lip and palate patients (Roberts et al., 1991). Three main parameters; number of patients, list of operations, and number of hospitals are generated along with other parameters such as hospital eligibility for different operations, hospital capacity, and hospital preference for patients (Testi et al., 2007). Each operation is scheduled on a weekly basis over a planning horizon (Jebali & Diabat, 2015). An example of surgical scheduling problem with 16 patients and 3 hospitals is used for model analysis. Each patient has 4 maximum operations. The planning period or time horizon is set as 28 weeks. Surgical capacity for hospitals is shown in Table 1.

Table 1. Surgery capacity for different hospitals.

Operation	Surgical capacity per week		
	Hospital 1	**Hospital 2**	**Hospital 3**
1	5	5	0
2	0	0	5
3	5	0	5
4	2	2	0

The problem is solved by exact method using LINGO optimization solver version 14.0. Figure 1 shows the optimal schedule of operations for all patients with assigned hospitals. The maximum completion time of all patients is 21 weeks and lease preference score are 81. The model yields an optimal solution with correct assignment and operation restrictions.

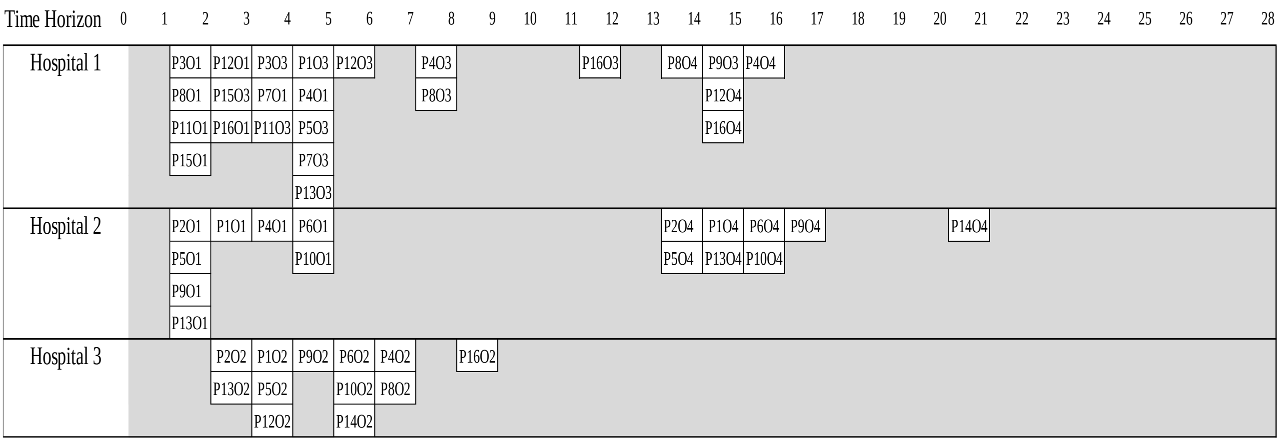

Figure 1. An optimal solution of the surgical scheduling problem.

4. Conclusions

In conclusion, there is an increasing demand for the use of Artificial Intelligence (AI) for smart decision, it is often done manually in practice. Since, surgical scheduling is a complex problem, the application of mathematical programming is used as a solution technique for decision making in surgical scheduling system. This use of mathematical model could pave the way to AI for smart decision of surgical scheduling in hospitals and help improving performance opportunities of this challenging problem. In this study, the multi-period surgical scheduling problem with limited surgery capacity is addressed. The goal is to schedule a list of patients who must undergo various kinds of operations by different eligible hospitals. Two objectives are considered in the proposed model; minimization of makespan and minimization of lease preference score of assigning patients to hospitals. The weighted sum approach is used to combine two objectives in a single objective.

Then, an instance inspired by real treatments of cleft lip and palate patients is generated in order to conduct computational experiment for model analysis. The result shows that the model yield the correct assignment and operation sequence respected to all constraints. This, the proposed mathematical programming model has potential to bring significant improvements to practice. Nevertheless, surgical scheduling problem is usually discussed under the assumptions that the surgery durations and capacity are deterministic variables, and only non-preemptive cases are considered. In practice, some of these assumptions are unrealistic. Hospital may be subjected to unpredictable conditions of their surgery capacity and surgery durations. Furthermore, arrival of emergency or urgent surgeries may occur and result in preemptions in the scheduling. Hence, the further research should be focused on stochastic modelling for handling the uncertainty in real-world practices.

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