

Original Article

Modeling and Optimization of Cost-Based Hybrid Flow Shop Scheduling Problem using Metaheuristics

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Abstract: The cost-based hybrid flow shop (CHFS) scheduling has been immensely studied due to its huge impact on productivity. For any profit-oriented organization, it is important to optimize total production costs. However, few researchers have studied hybrid flow shops (HFS) with total production cost utilization. This paper aims to develop a computational model and test the exploration capability of metaheuristics algorithms while optimizing the CHFS problem. Carlier and Neron defined three hypothetical benchmark problems for computational experiments. The popular optimization algorithms PSO, GA, and ACO were implemented on the CHFS model with ten optimization runs. The experimental results proven that ACO performed well regarding mean fitness value for all benchmark problems. Besides this, CPU time for PSO was very high compared to other algorithms. In the future, other optimization algorithms will be tested for the CHFS model, such as Teaching Learning Based Optimization (TLBO) and the Crayfish Optimization Algorithm (COA).

Keywords: Hybrid flow shop; Cost optimization; Metaheuristics.



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1. Introduction

Production scheduling is vital in manufacturing industries. Scheduling refers to assigning tasks using resources such as materials, machines, and humans to create valuable products. Production scheduling mainly has two categories, job shop and flow shop, depending on resource flow. One well-known variant is hybrid flow shop scheduling (HFS) (Ruiz & Vázquez-Rodríguez, 2010). HFS scheduling is having a set of stages with machines in parallel. The most critical problem with the Hybrid flow shop is finding optimal job sequences and assigning jobs to each machine at each stage. The automotive body press, printed circuit board assembly, chemical processing, textile, and metal processing sectors are all big HFS production scheduling system users. Efficient scheduling in the HFS problem reduces production costs and increases overall profitability. In any profit-oriented organization, cost is the main factor to be optimized to enhance overall profit. The HFS scheduling problem based on cost functions is called the cost-based hybrid flow

shop (CHFS) (Istokovic et al., 2020). It is a complex scheduling problem encountered in manufacturing and production environments.

Usually, manufacturing industries consider total production cost a major decision-making factor to optimize it. The associated costs, such as labor, electricity (Luo et al., 2013), maintenance, and delay, lead to the total production costs considered in this paper. In the literature, researchers implemented metaheuristics algorithms to optimize various CHFS problems (Janiak et al., 2007). Metaheuristic algorithms can solve complex combinatorial CHFS problems. As the number of jobs and machines increases, the number of potential solutions also increases. It became hard to manually calculate the best fitness function solution due to abundant solutions and time-consuming process. Although the metaheuristics are not guaranteed to give the exact solution, they only give near-optimal solutions. Numerous researchers have contributed to the field of CHFS, addressing a range of optimization objectives. The predominant research focus has been optimizing energy consumption and production costs tailored to specific requirements and situational contexts (Istokovic et al., 2020; Zheng et al., 2020). It has been observed that most researchers have focused on optimizing total production cost and energy cost (Geng et al., 2020; Istokovic et al., 2020), while considering makespan somehow in multi-objective optimization problems. Besides these costs, other costs are also considered, such as tardiness, inventory, and penalty costs, to be optimized (Dabiri et al., 2022; Sukkerd et al., 2021).

Other costs such as transportation, setup, rejected jobs, overtime, adjusting, operating, and resource allocation have also been considered (Behnamian & Fatemi Ghomi, 2011; Dabiri et al., 2022; Fei et al., 2010; Jiang et al., 2015; Moazami Goodarzi et al., 2021; Y. Wang et al., 2015; Zohali et al., 2019). Moving on, researchers have studied various CHFS variants. The basic version is known as identical parallel machines, where all machines are identical in every aspect on each stage (Anghinolfi et al., 2021). The second version is the unrelated parallel machine (UPM), where the parallel machines are independent of other machines (Fanjul-Peyro, 2020). The third version is a distributed hybrid flow shop (DHFS), where each job is processed by available plants/factories. Reentrant HFS is another variant that combines HFS with reentrant scheduling (Dong & Ye, 2022). Metaheuristic algorithms have been implemented for the optimization of CHFS problems. The most basic algorithms are named genetic algorithm (GA) (Brabazon et al., 2015), Particle Swarm Optimization (PSO) (Marini & Walczak, 2015) and differential evolutionary (DE) (Deng et al., 2021) algorithms, contributing most while optimizing CHFS problems observed in the literature.

Beside these algorithms, others were also used depending on the problem type and conditions. These algorithms are Ant colony optimization (Wang et al., 2016), Integer programming (IP) (Songserm & Wuttiornpun, 2019), Salp swarm algorithm (SSA) (Dong & Ye, 2022), Tabu search (TS) (Sukkerd et al., 2021), Iterated local search (ILS) (Zohali et al., 2019), Evolutionary algorithm (EA) (Lian et al., 2021), Memetic algorithm (MA) (Shao et al., 2022), Artificial bee colony (ABC) (Li et al., 2020), Heuristic algorithm (HA) (Fakhrzad & Heydari, 2008) and Ant lion optimization algorithms (ALO) (Geng et al., 2020). Even though there were a few publications on CHFS, most considered the cost individually. There is a lack of computational models considering the overall cost in CHFS. This paper proposes a comprehensive CHFS labor, electricity, maintenance, and penalty cost model. This paper presents a comprehensive overview of the modeling and optimization of CHFS. Section one overviews the introduction, followed by the CHFS model development. Lastly, the CHFS-developed model was tested for verification using a hypothetical dataset. Finally, the conclusion is presented based on the main findings.

2.2. CHFS Model Development

This section discusses the CHFS model development which consists of mathematical and computational models. The optimization objective of this work is to minimize the total production cost related to HFSS. Four costs are considered in this study: labor cost, electrical energy cost, preventive maintenance cost, and late penalty cost. Considering each cost separately to be discussed in terms of mathematical modeling. The objective functions for each cost are discussed below.

2.1. Mathematical Modeling

Labor Cost, C_L : Labor cost refers to the wage paid to the labor to perform a specific job for a particular period. In this work, C_L is assumed to be constant, taken from the average hourly pay rate. Another assumption is that only one laborer is required to operate one machine. The C_L is calculated by multiplying all machines' total operation time and the hourly pay rate. The default time unit for operation time is in minutes. Where j is the number of jobs, S is the total number of stages, and M represents the number of

machines. Also $t_{j sm}$ is processing time of job J on machine M at stage S in minutes. The term $\alpha_{j sm}$ is a binary variable with constraints in equation (1).

$$C_L = \left(\sum_{j=1}^J \sum_{s=1}^S \sum_{m=1}^M t_{j sm} \cdot \alpha_{j sm} \right) \times \left(\frac{\text{Hourly payrate}}{60} \right) \tag{1}$$

$$\alpha_{j sm} = \begin{cases} 1, & \text{if job } j \text{ is processed on machine } m \text{ at stage } s \\ 0, & \text{otherwise} \end{cases}$$

Electricity Cost, C_E : The electricity cost is only the electricity used to operate the machines. Thus, the electricity used for other purposes, such as lighting and ventilation, is not considered. The machines' electricity cost during idle or standby mode is also not considered. The electricity cost is calculated from the power consumption of a particular machine. The machine's power rate will be multiplied by the operation duration and converted into a kWh unit. This work considers non-identical machines; thus, the power rate for the machines in a similar stage might differ, ultimately affecting the total power utilization. In Equation (2), the first term represents total energy utilization in watt minutes. The second term converts the energy utilization into kWh and multiplies it with the average electricity tariff for the total energy cost. Where p_{sm} is the power rating of machines in watts.

$$C_E = \left(\sum_{j=1}^J \sum_{s=1}^S \sum_{m=1}^M t_{j sm} \cdot p_{sm} \cdot \alpha_{j sm} \right) \times \left(\frac{\text{Average electricity tariff}}{60 \times 1000} \right) \tag{2}$$

Maintenance Cost, C_R : Maintenance cost is the cost to ensure the assets are in good working condition. Typically, it can be classified into preventive and corrective maintenance. In this work, the maintenance cost is limited to usage-based preventive maintenance for the machines. The maintenance will be performed based on the predetermined maximum operating duration for machines. Each machine's operating duration and maintenance cost will differ depending on the model. The number of maintenances required is calculated by rounding up the result of the division of the total operation time of machine m at stage s (T_{sm}) and the maximum operating duration of machine m at stage s , tr_{sm} as shown in Equation (3).

$$C_R = \sum_{s=1}^S \sum_{m=1}^M \left\lceil \frac{T_{sm}}{tr_{sm}} \right\rceil \times r_{sm} \tag{3}$$

Late Penalty Cost, C_P : Late penalty is imposed on the manufacturer due to failure to deliver the orders within the agreed period. In this work, the late penalty will be implemented if the manufacturer fails to complete the requested quantity by the due date. The late penalty will be imposed daily. A longer overdue job will cause a larger late penalty cost. Where C_j is completion date and D_j is the due date of jobs to be delivered. The term Y_j is known as the lateness factor presented in equation (4), while C_P is the total late penalty cost and can be calculated using equation (5).

$$Y_j = \left(\frac{C_j}{\text{Working time per day in minutes}} \right) - D_j \tag{4}$$

$$Y_j = \begin{cases} y_j & \text{if } y_j > 0 \\ 0 & \text{else} \end{cases}$$

$$C_P = \sum_{j=1}^J Y_j \times \text{Daily penalty charges} \tag{5}$$

2.2. Fitness Function

In our problem the fitness function encounters the total production cost. The fitness function within our computational model has been intentionally built to accurately measure the cost-effectiveness and efficiency of production scheduling for the CHFS problem. Once the fitness function has been designed for the problem, later it is then linked with various metaheuristics algorithms for optimization. The algorithm which is best suited for the problem is declared as the best optimization algorithm. In our case there is a single criterion that needs to be optimized, which is total production cost. The total production cost as a single fitness function captures four major costs (labor cost, electricity cost, preventive maintenance cost and late penalty cost). Equation (6) presents fitness function formula below.

$$\text{Min } f(x) = C_L + C_E + C_R + C_P \quad (6)$$

Where labor cost is the cost associated with all jobs processing time which is then converted to a quantitative value by multiplying it with hourly pay rate. The electricity cost is determined in terms of machine power rating and average electricity tariff. Maintenance cost is calculated based on maintenance cost of machines and the maintenance cycles. Finally the late penalty cost is related to when jobs are finished after their due date multiplied by penalty rate. Once all costs are calculated, fitness comes out with total production cost.

3. Numerical Example

A simple numerical example has been considered for the CHFS model with six jobs and two stages. Table 1 presents the processing times for each job on various machines in both stages, along with information about maintenance costs, mean time to repair, and power ratings for each machine. Additionally, Table 1 also depicts the proper assumptions that have been considered.

Table 1. Data and Assumptions for Computational Model

| Job | Stage 1 | | Stage 2 | | Job Due |
|------------------------|------------------|------------------|------------------|------------------|---------|
| | M ₁₋₁ | M ₁₋₂ | M ₂₋₁ | M ₂₋₂ | |
| J1 | 5 | 15 | 7 | 11 | 80 |
| J2 | 3 | 19 | 9 | 14 | 80 |
| J3 | 3 | 18 | 12 | 9 | 48 |
| J4 | 3 | 19 | 15 | 8 | 96 |
| J5 | 7 | 9 | 17 | 8 | 192 |
| J6 | 18 | 20 | 10 | 12 | 96 |
| MTTR | 120 | 150 | 140 | 80 | |
| Maintenance Cost | 200 | 200 | 200 | 120 | |
| Power Rating(kw) | 1.69 | 1.56 | 1.78 | 1.84 | |
| Assumptions: | | | | | |
| Cost Per hour | MYR12 | | | | |
| Penalty Charges | MYR5 | | | | |
| Avg Electricity Tariff | 0.65MYR/kwh | | | | |
| Job Sequence | [2 4 3 1 5 6] | | | | |

To calculate the fitness function (total production cost) manually the following steps are as follows. Where total production cost refers to four major costs (Labor cost, Electricity cost, Maintenance cost, Late Penalty cost) captured in this CHFS problem. Step 1. Labor Cost, C_L : In the first step the labor cost is calculated in terms of total machine processing time (tmpt). The (tmpt) is formulated by summing all machine processing times (mpt) divided by 60 and is then multiplied with the hourly pay rate to find the labor cost. In this problem the hourly pay rate is taken as 12MYR per hour as shown in Table 1. The expression to determine the total labor cost (TLC) is displayed in equation (7).

$$\text{TLC} = \text{tmpt} * \text{Hourly pay rate} \quad (7)$$

Step 2. Electricity Cost, C_E : The total electricity cost (TEC) is evaluated based on the total machines' power consumption and average electricity tariff. The total power consumption is expressed when (tmpt) multiplied with all machines power rating divided by 60 and is depicted in equation (8).

$$TEC = Total\ power\ consumption * Average\ electricity\ tariff / 60 \quad (8)$$

Step 3. Maintenance Cost, C_R : The cost of maintenance is determined when maintenance cycles (MT) are multiplied elementwise with maintenance cost (RR) of each machine. The find maintenance cycles (MT), (tmpt) are divided elementwise with mean time to repair (MTTR) and round off result. The formula used to calculate the maintenance cost is presented in equation (9).

$$MC = MT.* RR \quad (9)$$

Step 4. Late Penalty Cost, C_P : Finding delay cost is dependent on jobs delay duration and penalty charges. The penalty rate is 5MYR per minute in this CHFS problem. Delay can be found with the difference between finish times (FT) and due times (DT) of jobs. To quantify the total delay cost (TDC) equation (10) depicts the final expression.

$$TDC = Delay * Penalty\ charges \quad (10)$$

Finally, to calculate the total production cost (TPC) is illustrated in equation (11).

$$TPC = TLC + TEC + MC + TDC \quad (11)$$

The Pseudo-code to calculate the fitness function is presented below.

Input: Initialize Job Time Matrix, No of Jobs, No of Machines at each stage, No of Stages, No of Machines, Generate Random Vector x and initialize xc Matrix

Output: Total Production Cost, TPC

```

for nstage and njob
    Find mach_assign and pt_assign {Find machine assignment and processing time of job}
end for
for njob calculate mn2
    Sort njob in ascending order of mn2 {Identify job processing sequence}
end for
for j = 1: njob
    for m = 1: nmachine
        read mach_due for mth machine (mach_due)
        read prec_due for jih job (prec_due,)
        read (pt)jm from processing_time {Processing time for jobs on machines}
    end for
end for
if mach_due > prec_due
    es = prec_due {Earliest start time for jth job}
else
    es = mach_duem
    ft = es + pt {Finish time for jobs}
    update mach_due = ft
    update prec_due
end if
for njob in the sorted sequence find ST and FT {Considering constraints calculate start and finish times of jobs}
    Calculate makespan Cmax
end for
for nstage and mach_num calculate (mpt) {Calculate machine processing time}
    calculate (tmpt) and CL {Calculate total machine processing time and labor cost}
end for
for nmach define power rating and avg electricity tariff
    calculate total power consumption
    calculate TEC {Calculate the total electricity cost}
end for
for nmach define MTTR, RR, and (tmpt) {Define mean time to repair, maintenance cost and total machine processing time}
    calculate (MT) and (MC) {Find maintenance cycles total maintenance cost}
    
```

```

end for
for njob calculate (FT-DT) {Calculate delay for jobs}
    if delay<0 delay=0
        else delay > 0
            calculate total delay cost (DC)
    end for
end if
    
```

4. Optimization of CHFS Problems Using Metaheuristics

This section describes the computational experiment for the optimization of the CHFS problem with particle swarm optimization (PSO), Genetic Algorithm (GA), and Ant colony optimization (ACO). The purpose of the computational experiment is to verify the efficiency of the proposed algorithm while optimizing CHFS problems.

4.1. Computational Experiment

A computational experiment has been conducted for CHFS problems to test the proposed algorithms for various problem dimensions. For this purpose, a set of benchmark problems proposed by [Carlier & Neron \(2000\)](#) was utilized ([Carlier & Neron, 2000](#)). This well-known hypothetical scenario is frequently used to evaluate hybrid flow shop scheduling problems. The benchmark test problem is demonstrated in Table 2. The processing time is randomly created using normal distribution in the range of {3 20}. Machine configuration in Table 2 refers to the number of machines at each stage.

Table 2. Benchmark Test Problems Configuration

| Problem | No. of Jobs | Number of Stages | Machine Configurations |
|---------|-------------|------------------|------------------------|
| J10c5a2 | 10 | 5 | 2 2 1 2 2 |
| J10c5b1 | 10 | 5 | 1 2 2 2 2 |
| J10c5c1 | 10 | 5 | 3 3 2 3 3 |

The performance of popular metaheuristics algorithms is to be compared to get the best optimal solutions for each problem. PSO is a well-known optimization algorithm mostly used in hybrid flow shop problems. Besides this, GA is also utilized in many hybrid flow shops with various cost utilization. Finally, the ACO will be tested, and the results will be compared. The population size in the computational experiment was set up to 50, with a maximum iteration of 1000 for all algorithms. The optimization run will be set to 10 times for each benchmark problem. The averages of fitness values, maximum and minimum values, along with their respective standard deviations, are shown in Table 3.

Additionally, the CPU times for each optimization problem are provided for various algorithms. Overall, the ACO algorithm gives the best fitness value in all three problems. Meanwhile, GA ranked second in the mean fitness value throughout all three problems. Besides this, the PSO was found with the larger mean fitness solution among all three benchmark problems. In addition, the computational time (CPU time) for PSO is relatively larger compared to ACO and GA. However, CPU time for GA and ACO have a smaller difference. For optimization runs of value 10 in the case of problem j10c5c1, the ACO algorithm achieved the minimum fitness value across the three problems. For the problems J10c5a2 and J10c5b1, during 10 optimization runs, the PSO algorithm showed superior performance over GA to achieve the minimum fitness value. The results show that ACO is best suited for convergence in every optimization run.

Table 3. Fitness Function Mean, Standard Deviation, Maximum, and Minimum Value with CPU Time

| Problem | Indicator | PSO | GA | ACO |
|---------|-----------|----------|----------|----------|
| J10c5a2 | Mean | 1042.215 | 1082.045 | 873.9249 |
| | SD | 258.5666 | 114.0036 | 15.51468 |
| | Max | 1596.147 | 1234.426 | 910.9665 |
| | Min | 776.9386 | 939.9875 | 864.1301 |
| | CPU time | 922.8019 | 54.00155 | 46.11876 |
| J10c5b1 | Mean | 789.1552 | 976.6594 | 705.0356 |
| | SD | 199.3359 | 159.568 | 84.24768 |
| | Max | 1085.106 | 1175.756 | 943.6938 |
| | Min | 566.9447 | 650.4568 | 657.4353 |

| Problem | Indicator | PSO | GA | ACO |
|---------|-----------|----------|----------|----------|
| J10c5c1 | CPU time | 1078.904 | 62.51528 | 45.47489 |
| | Mean | 449.7107 | 460.684 | 440.0794 |
| | SD | 6.386267 | 7.52624 | 4.314444 |
| | Max | 460.7808 | 472.066 | 446.9412 |
| | Min | 441.3033 | 450.111 | 433.1657 |
| | CPU time | 1246.97 | 85.5354 | 58.88473 |

4.2. Verification of CHFS Computational Model

The computational model has been verified with manual calculation. For the computational model to be verified, the computational results must match the manual calculation results. The simple numerical example data for manual calculation is shown in Table 1. There are four main parameters in the CHFS model to be calculated step by step. First, makespan, next labor cost, Electrical energy cost, moving on to the maintenance cost and finally, a late penalty cost will be calculated. The simple data in Table 1 will be utilized while calculating these parameters. The data needs to be organized to find the makespan, including the job sequence, to determine the output table. Makespan is the maximum completion time for all jobs. The random job scheduling sequence is based on the shortest processing time (SPT) at stage one, as shown in Table 1. The stepwise calculation for the makespan is as follows (see Table 4 and 5).

Table 4. Step1: Assign Jobs to Machines at Stage 1

| Job Seq | S2M1 (M21) | S2M2 (M22) | ST | FT |
|---------|------------|------------|----|----|
| 2 | 3 | 19 | 0 | 3 |
| 4 | 3 | 19 | 3 | 22 |
| 3 | 3 | 18 | 22 | 25 |
| 1 | 5 | 15 | 25 | 40 |
| 5 | 7 | 9 | 40 | 47 |
| 6 | 18 | 20 | 47 | 65 |

Table 5. Step 2: Assign Jobs to Machines at Stage 2

| Job Seq | S2M1 (M21) | S2M2 (M22) | ST | FT |
|---------|------------|------------|----|----|
| 2 | 9 | 14 | 3 | 17 |
| 4 | 8 | 15 | 22 | 37 |
| 3 | 12 | 9 | 25 | 36 |
| 1 | 7 | 11 | 40 | 47 |
| 5 | 17 | 8 | 47 | 55 |
| 6 | 10 | 12 | 65 | 75 |

To calculate the makespan, the maximum value of finish time on stage 2 will be considered. The makespan value is evaluated at 75 in output Table 6.

Table 6. Output Table for Jobs Sequence [2 4 3 1 5 6]

| Job | Stage | Machine | PT | ST | FT |
|-----|-------|---------|----|----|----|
| 2 | 1 | 1 | 3 | 0 | 3 |
| 2 | 2 | 2 | 14 | 3 | 17 |
| 4 | 1 | 2 | 19 | 3 | 22 |
| 4 | 2 | 2 | 15 | 22 | 37 |
| 3 | 1 | 1 | 3 | 22 | 25 |
| 3 | 2 | 1 | 12 | 25 | 36 |
| 1 | 1 | 2 | 15 | 25 | 40 |
| 1 | 2 | 1 | 7 | 40 | 47 |
| 5 | 1 | 1 | 7 | 40 | 47 |
| 5 | 2 | 2 | 8 | 47 | 55 |

| | | | | | |
|---|---|---|----|----|---------------|
| 6 | 1 | 1 | 18 | 47 | 65 |
| 6 | 2 | 1 | 10 | 65 | 75 = Makespan |

Moving on to manual calculations for associated costs, data from Table 1 will be utilized.

1. Labor Cost Manual Calculation, C_L : To calculate labor cost manually, the first total machine processing time (tmpt) has been calculated by summing all processing times of all machines from Table 4. Hence (tmpt) came out 131minutes. Finally total labor cost is determined using equation (7).

$$TLC = \text{tmpt} / 60 * \text{Hourly pay rate}$$

$$TLC = 131 / 60 * 12$$

$$TLC = 26.2 \text{ MYR}$$

2. Electricity Cost Manual Calculation, C_E : Electricity cost manual calculations can be done by utilizing equation (8). Initially power consumption is calculated for each machine and is then added for total power consumption. Thereafter total electricity cost (TEC) is calculated as follows.

$$\text{Power Consumption} = \text{Machines Processing time.} * \text{Machine Power Rating}$$

$$\text{Power Consumption} = [65 \quad 37 \quad 50 \quad 52]. * [1.69 \quad 1.56 \quad 1.78 \quad 1.84]$$

$$\text{Power consumption} = [109.85 \quad 57.72 \quad 89 \quad 95.68]$$

$$\text{Total power consumption of all machines} = \text{sum (power consumption)} = 352.12 \text{KW-minutes}$$

$$TEC = 352.12 * 0.65 / 60$$

$$TEC = 3.814416667 \text{MYR}$$

3. Maintenance Cost Manual Calculations, C_R : Maintenance costs is manually calculated by using equation (9). At first maintenance cycles are calculated using the formula ($MT = \text{round}(\text{mpt.} / \text{MTTR})$) with data utilization from Table 1. After that maintenance cost is calculated as follows.

$$MT = \text{round} [(65 \quad 37 \quad 50 \quad 52). / (120 \quad 150 \quad 140 \quad 80)]$$

$$MT = \text{round} [0.54 \quad 0.24 \quad 0.35 \quad 0.65] = [1 \quad 0 \quad 0 \quad 1]$$

$$MC = [1 \quad 0 \quad 0 \quad 1]. * [200 \quad 200 \quad 200 \quad 120]$$

$$MC = \text{Sum (MC)}$$

$$MC = 320 \text{MYR}$$

4. Late Penalty Manual Calculations, C_p : Delay cost manual calculations are carried out by utilizing data from Table 1. At the beginning, a delay is calculated by subtracting due times from the finish times of jobs and is then multiplied by the penalty rate, which is mentioned as 5MYR per minute. The stepwise manual calculations are given below to find the total delay cost. A negative penalty cost will be considered zero. It means the jobs are delivered before due time.

$$\text{Job-Due} = [80 \quad 80 \quad 48 \quad 96 \quad 192 \quad 96]$$

$$\text{Job-finish-times} = [47 \quad 17 \quad 36 \quad 37 \quad 55 \quad 75]$$

$$\text{Delay} = \text{Finish time} - \text{Due time} = [-33 \quad -63 \quad -12 \quad -59 \quad -137 \quad -21]$$

$$\text{Delay Cost} = \text{Delay} * \text{penalty charges} = [-33 \quad -63 \quad -12 \quad -59 \quad -137 \quad -21] * 5 \text{MYR}$$

$$\text{Delay Cost} = [-165 \quad -315 \quad -60 \quad -295 \quad -685 \quad -105]$$

$$\text{Total Delay Cost} = \text{Sum (Delay Cost)} = -1625 \text{MYR}$$

$$\text{TDC} = 0 \text{MYR}$$

Once all costs are calculated, the total production cost (TPC) is calculated by using equation (11). $TPC = 26.2 + 3.8146 + 320 + 0 = 350.0146 \text{MYR}$. All costs have been calculated manually. It has been observed that all results came out almost correct from the computational model compared to manual calculation. To verify that the output from the MATLAB code matches the manual calculations, the screenshot from the output has been depicted in Figure 1 for strong evidence.


```
Command Window
Makespan: 75
Total Labor Cost: MYR 26.2
Total Electricity Cost for All Machines:MYR 3.816
Total Maintenance Cost for All Machines: MYR 320
Total Delay Cost: MYR 0

Total_Cost =
    350.0160
fx >> |
```

Figure 1. Output from MATLAB Code

5. Conclusions

In conclusion, this paper aims to establish a computational model of a cost-based hybrid flow shop (CHFS) scheduling problem that can be used to simulate and optimize CHFS problem. In contrast with most existing CHFS problems, this study considered the total production cost comprised of labor cost, electricity cost, maintenance cost, and late penalty cost as one objective function for better decision-making in profit-oriented organizations. The research starts to develop a computational model for CHFS. Later, the model feed with the simple data is taken for manual calculation for model verification. Next, a computational experiment was conducted using the three most used optimization algorithms: PSO, ACO, and GA. The results were recorded for 10 optimization runs, and various indicators were assessed. The computational experiment shows that ACO performance is best for mean fitness value in all problems; PSO ranked second and last GA. The results indicated the applicability of the proposed computational model to standard metaheuristic algorithms. Manual calculation proves that the model is accurate in representing the CHFS problem.

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